

# Hysteresis and spin phase transitions in quantum wires in the integer quantum Hall regime

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We demonstrate that a split-gate quantum wire in the integer quantum Hall regime can exhibit electronic transport hysteresis for up- and down-sweeps of a magnetic field. This behavior is shown to be due to phase spin transitions between two different ground states with and without spatial spin polarization in the vicinity of the wire boundary. The observed effect has a many-body origin arising from an interplay between a confining potential, Coulomb interactions and the exchange interaction. We also demonstrate and explain why the hysteretic behavior is absent for steep and smooth confining potentials and is present only for a limited range of intermediate confinement slopes.

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*Introduction.* The effect of hysteresis represents fundamental phenomena occurring in a variety of systems ranging from conventional metallic ferromagnets [1] to quantum resonant devices [2]. The hysteresis effect is usually associated with bistable behavior where a system can undergo phase transitions or can have different ground states. Recently, a new phenomenon of electronic transport hysteresis has been observed in two-dimensional electron gas systems (2DEG) including double-layered quantum well structures and conventional (one-layered) modulation-doped heterostructures both in the regimes of the integer and the fractional quantum Hall effects (IQHE and FQHE) [3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. The origin of the hysteretic behavior in the double-layered structure was attributed to the phase transition between two oppositely polarized ground states localized in different layers [3]. The observations of hysteretic behavior in the modulation-doped heterostructures in the FQHE suggest the presence of a novel two-dimensional ferromagnetism where the spin-polarized ground state competes with spin-unpolarized one [4, 5, 6], even though the detailed many-body origin of the observed effect still remains unclear. In contrast, the hysteresis reported in [7] in the IQHE regime does not seem to require a many-body explanation and can be understood in terms of a co-existence and a dynamical exchange of electrons due to the presence a parallel conducting channel.

Theoretical investigations of hysteretic phenomena in 2DEG systems have received less attention [13, 14, 15, 16], being mostly limited to periodically-modulated systems where hysteresis was observed typically by varying the strength of the modulation (as opposed to up- and down-sweeps of the magnetic fields used in the experiment).

In this Letter we predict transport hysteretic behavior for a quantum wire geometry in the IQHE regime for up- and down-sweeps of the magnetic field. We demonstrate that this behavior is related to the transition between

two phases corresponding respectively to two different ground states with spatially spin-polarized and spatially spin-unpolarized edge channels in the vicinity of the wire boundary. We also demonstrate and explain why the hysteretic behavior is absent for steep and smooth confining potentials and is present only for a limited range of intermediate confinement slopes. The IQHE regime is usually associated with a one-electron description. We stress that the observed effect has a many-body nature arising as an interplay between a confining potential, the Coulomb interaction and the exchange interaction.

We also note that the predicted hysteretic behavior not only sheds new light on the structure of the edge states and spin transitions in the IQHE regime. The predicted effect can occur in the leads of lateral nanostructures, which might have important consequences for the magnetotransport of all lateral quantum devices, particularly, for those exploiting spin polarized injection and detection by means of the spatial separation of spins [12, 17].

In order to incorporate electron interaction and spin effects we use the density functional theory (DFT) in the local spin density approximation [18, 19]. The validity of this approximation for the 2DEG systems is supported by the excellent agreement with the exact diagonalization and variational Monte-Carlo calculations performed for few-electron systems [20]. An important feature of our approach is that we start with a lithographical geometry of the device (see Fig. 1 (a)) and do not use any phenomenological parameters (such as charging constants, coupling strengths etc.). In this respect, it is important to stress that the results of the DFT-based transport modelling indicate that utilization of the simplified approaches using phenomenological parameters and/or model Hamiltonians might not always be reliable for theoretical predictions as well as interpretations of experiments [21].

*Model.* We consider a quantum wire in a perpendicular magnetic field described by the Hamiltonian  $H =$

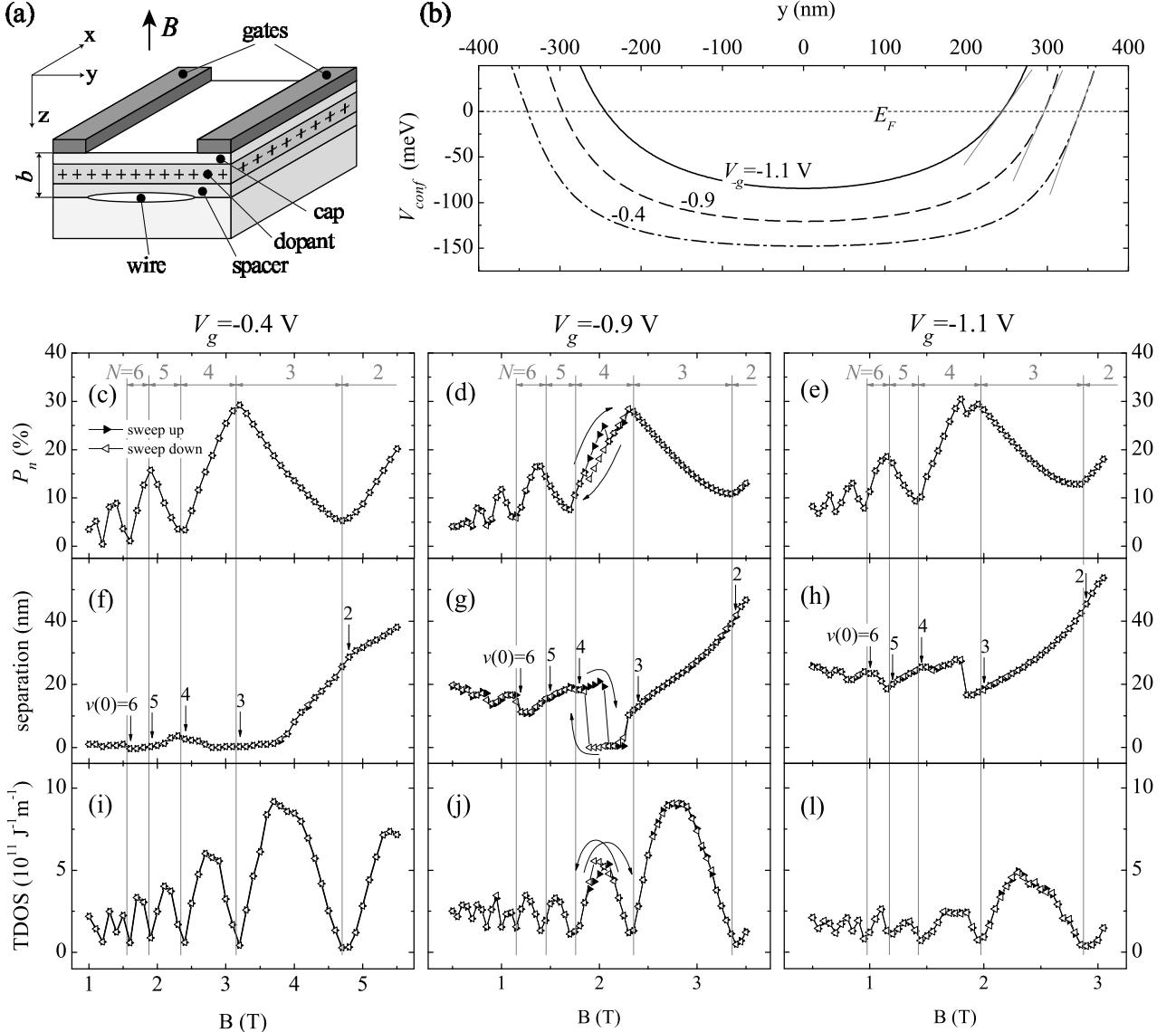


FIG. 1: (a) A schematic layout of a split-gate quantum wire in perpendicular magnetic field. Parameters of the wire: the distance between gates  $w = 700$  nm, the thickness of donor layer  $d = 26$  nm, the cap layer  $c = 14$  nm, the distance between gates and the two-dimensional electron gas (wire)  $b_{2DEG} = 50$  nm; the dopant concentration  $1 \cdot 10^{24} \text{ m}^{-3}$ . (b) Electrostatic potentials for the quantum wire for three gate voltages:  $V_g = -0.4$  V, -0 (solid line),  $V_g = -0.9$  V (dashed line),  $V_g = -1.1$  V (dash-dotted line). The tangents to the potential curves at  $E_F$  indicate the steepness of the potential near the boundary. (c)-(e) The charge density spin polarization, (f)-(h) the spatial separation between the states  $N = 1$  and 2, and (i)-(l) the thermodynamical density of states for three gate voltages from Fig. 1 (b).  $N$  and  $\nu(0)$  indicates the number of the occupied subbands and the local filling factor in the middle of the wire. Temperature  $T = 1$  K.

$\sum_\sigma H^\sigma$ ,  $H^\sigma = H_0 + V^\sigma$ , where  $H_0$  is the kinetic energy in the Landau gauge,  $H_0 = -\frac{\hbar^2}{2m^*} \left\{ \left( \frac{\partial}{\partial x} - \frac{eiBy}{\hbar} \right)^2 + \frac{\partial^2}{\partial y^2} \right\}$ , and  $m^* = 0.067m_e$  is the GaAs effective mass, see Fig. 1 (a). The total confinement potential  $V^\sigma$  for the spin-up ( $\sigma = +\frac{1}{2}, \uparrow$ ) and spin-down ( $\sigma = -\frac{1}{2}, \downarrow$ ) electrons reads

$$V^\sigma = V_{conf}(y) + V_H(y) + V_{xc}^\sigma(y) + g\mu_b B\sigma, \quad (1)$$

where  $V_{conf}(y)$  is the bare electrostatic confinement including contributions from the Schottky barrier

$V_{Schottky} = 0.8$  eV, the split-gates [22] and the dopant layer [23] (for the explicit expressions see [24]). The Hartree potential due to the electron density  $n(y) = \sum_\sigma n^\sigma(y)$  (including the mirror charges) reads [24]

$$V_H(y) = -\frac{e^2}{4\pi\varepsilon_0\varepsilon_r} \int dy' n(y') \ln \frac{(y - y')^2}{(y - y')^2 + 4b^2}, \quad (2)$$

where  $\varepsilon_r$  is the relative permittivity for GaAs  $\varepsilon_r = 12.9$  and  $b$  is the distance to the surface.  $V_{xc}^\sigma(y)$  is the exchange-correlation potential included within the frame-

work of the Kohn-Sham density functional theory [18] in the local spin-density approximation using the parameterization of Tanatar and Cerperly [19] (see Ref. 24 for explicit expressions for  $V_{xc}(y)$ ). The last term in Eq. (1) accounts for the Zeeman splitting with  $\mu_b = \frac{e\hbar}{2m_e}$  being the Bohr magneton and the bulk  $g$  factor of GaAs,  $g = -0.44$ . The spin-resolved electron density is given by the retarded Green's function,  $G^\sigma(y, y, E)$ ,  $n^\sigma(y) = -\frac{1}{\pi} \Im \int dE G^\sigma(y, y, E) f(E - E_F)$ , where  $f(E - E_F)$  is the Fermi-Dirac distribution function. The Green's function of the wire as well as the electron and the current densities are calculated self-consistently using the technique described in detail in Ref. 24. We calculate also the thermodynamical density of states (*TDOS*) that reflects the structure of magnetosubbands near the Fermi energy [25],

$$TDOS = \sum_\sigma \int dE \rho^\sigma(E) \left( -\frac{\partial f(E - E_F)}{\partial E} \right), \quad (3)$$

where the density of states  $\rho^\sigma(E) = -\frac{1}{\pi} \Im \int dy G^\sigma(y, y, E)$  [26].

*Results and discussion.* Figure 1 shows the spin polarization of the electron density,  $P_n = \frac{n_{1D}^\uparrow - n_{1D}^\downarrow}{n_{1D}^\uparrow + n_{1D}^\downarrow}$  ( $n_{1D}^\sigma$  is  $\int dy n^\sigma(y)$  is the one-dimensional density), the spatial separation between the innermost (closest to the wire boundary) edge states corresponding to the spin-up and spin-down subbands  $N = 1, 2$  and the *TDOS* for three representative gate voltages. The density spin polarization  $P_n$  shows a distinct  $1/B$ -periodic looplike pattern, whose periodicity is related to the subband depopulation (for a detailed analysis of various aspects of the spin polarization and magnetosubband- and edge state structure in quantum wires see Refs. [24, 27, 28]). In the present Letter we concentrate at a pronounced hysteretic behavior for forward- and reverse magnetic field sweeps as shown in Fig. 1 (d),(g),(j) for the gate voltage  $V_g = -0.9$  V. Note that the hysteretic behavior is present only for some intermediate range of the gate voltage and is absent for lower and larger voltages, see Fig. 1 (c),(f),(i) and (e),(h),(l).

The key to understanding of the observed hysteresis effect is the spatial polarization of the innermost edge states shown in Fig. 1 (g). For the magnetic field  $B \lesssim 1.8$  T (corresponding to the filling factor  $\nu \gtrsim 4$ ), the spatial separation  $d$  between  $N = 1$  (spin-up) and  $N = 2$  (spin-down) states is almost independent of  $B$  and is nearly constant. This spatial polarization is due to the lifting of the spin degeneracy by the exchange interaction for the case of a sufficiently smooth potential confinement [29]. For the magnetic field  $B \gtrsim 2.1$  T ( $\nu \lesssim 3$ ) the spatial separation  $d$  gradually increases. This behavior of  $d$  can be traced back to the evolution of the compressible strips in the Chklovskii *et al.* [30] model of the spinless electrons, where the width of these strips,  $w_{comp}$ , monotonically increases with increase of  $B$ . The exchange interaction is

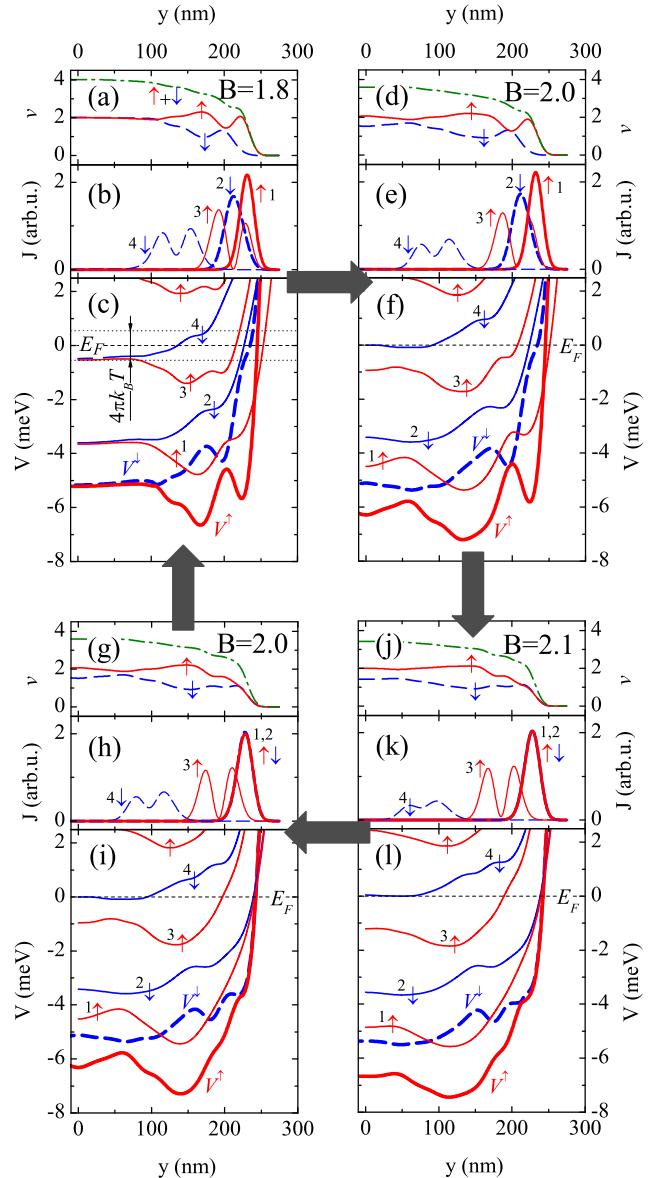


FIG. 2: The electron density profile,  $\nu = \frac{n(y)}{n_B}$ , current densities and magnetosubband structure for  $V_g = -0.9$  V and four values of magnetic field: (a)-(c) -  $B = 1.8$  T, (d)-(f) -  $B = 2.0$  T (sweep up), (g)-(i) -  $B = 1.9$  T (sweep down), (j)-(k) -  $B = 2.1$  T. Big arrows show the sweeping direction. Fat solid lines indicate the total confining potentials  $V^\sigma$ ; the integers 1-4 indicate the subband number. Temperature  $T = 1$  K.

shown to affects dramatically the compressible strips by suppressing them and inducing the spatial separation of the spin states  $d \approx w_{comp}$  [27].

In the field interval  $1.8 \text{ T} \lesssim B \lesssim 2.1$  T, the quantum wire exhibits a bistable behavior with two distinct ground states with and without the spatial spin polarization. The origin of the ground state with the suppressed spatial spin polarization can be understood from an evolution of the band structure of the quantum wire.

For further analysis it is important to emphasize that the Coulomb energy is dominant for the system at hand such that the total electron density distribution  $n(y)$  is practically unaffected by the applied magnetic field [30]. Figures 2 (a)-(c) show the band structure and the current and electron densities for  $B = 1.8$  T. The innermost spin-polarized states (denoted by 1 and 2 in (b)) are spatially separated due to the exchange interaction. The spin-up state 3 belonging to the outermost subband partially overlaps with the states 1 and 2. As the magnetic field increases the 3rd subband is pushed up in the energy. As a result, the corresponding electron density is redistributed towards the center of the wire, see Fig. 2 (e) for  $B = 2$  T, and the overlap between the states 2 and 3 increases. As the state 3 moves away from the boundary, the density of the remaining electrons has to be adjusted to keep the total density  $n(y)$  unchanged. This can be done only if the spin-down electrons associated with the 2nd subband are redistributed towards the edge of the wire. (Note that the Coulomb interaction is much stronger than the exchange interaction separating states 1 and 2). This redistribution leads to the collapse of the spatial spin separation, see Fig. 2 (k) for  $B = 2.1$  T. This phase of the system with the ground state without the spatial spin polarization is preserved up to  $\nu \approx 3$  for the sweep of the magnetic field in the forward direction. For a higher field (corresponding to the formation of compressible strips in the Chklovskii *et al.* model of spinless electrons) the exchange interaction restores the spatial spin separation of the order  $d \approx w_{comp}$  [27] as discussed above.

For the field sweep in the reverse direction, the spatial spin polarization is restored for the magnetic field significantly lower ( $\sim 1.8$  T) than that when the transition from the spin-polarized to the spin-unpolarized phase takes place for the forward sweep ( $\sim 2.1$  T). The system shows a memorization of the spin polarization, similar to a memorization of the magnetization direction in ferromagnetic domains. Thus, in the above interval ( $1.8 \text{ T} \lesssim B \lesssim 2.1$  T), the quantum wire exhibits a bistable behavior where the system, depending on the history, can be in one of the ground states (with or without the spatial spin polarization). The bistable behavior of the quantum wire is manifest itself in the spin polarization of the electron density  $P_n$  (Fig. 1 (d)) and the TDOS (Fig. 1 (j)). The latter can be accessible via magneto-capacitance [31] or magnetoresistance [32] measurements. The spin polarization of the electron density  $P_n$  can be probed directly using e.g. polarized photoluminescence spectra as recently reported by Nomura and Aoyagi [33]. The predicted hysteretic behavior can also be probed in transport measurements on lateral quantum dots in the edge state regime involving spin polarized injection and detection by means of the spatial separation of spins [12, 17].

Let us now discuss a condition for the hysteretic behavior to occur. We demonstrate below that the key

factor affecting the hysteresis is a steepness of the electrostatic confinement near the wire boundaries. [Note that application of more negative gate voltage results in a potential confinement with a less steep slope near the wire boundaries, see Fig. 1 (b)]. Figure 1 (f) shows the spatial separation  $d$  for the applied voltage  $V = -0.4$  V. In the magnetic field region  $3 \lesssim \nu \lesssim 4$  (when the hysteresis is present for  $V = -0.9$  V), the spatial spin separation is practically absent. This is because of a steep slope of the potential profile such that the strength of the exchange interaction is not enough to pull the spin species apart [29]. As a result, *a phase corresponding to the spatially polarized ground state is not present in wires with the steep walls and the bistable behavior is not possible*. Opposite situation occurs for the case of a smooth confinement,  $V = -1.1$  V, see Fig. 1 (h), when the ground state can be only spatially polarized in the above field interval ( $3 \lesssim \nu \lesssim 4$ ). An inspection of the corresponding band structure calculated in the Hartree approximation for the spinless electrons (which well corresponds to the Chklovskii *et al.* model [27, 34]) reveals an onset of a formation of the compressible strips. [The compressible strips are more easily formed in a wire with more smooth confinement [30]]. As discussed above, the compressible strips are suppressed by the exchange interaction [27], which leads to the spatial separation of the spin states  $d \approx w_{comp}$ . *This results in the absence of the spin-unpolarized phase (and thus the hysteretic behavior) in wires with a smooth confinement*.

It should be stressed that the results and conclusions reported above are neither specific nor limited to the particular parameters of the quantum wire of Fig. 1. Similar hysteretic behavior has been detected in different quantum wires. Note that in some cases a less pronounced hysteresis (of the same origin as above due to the collapse of the spatial spin separation) has been also seen in a vicinity of  $\nu \approx 6$ . The parameters of the quantum wire considered in this Letter have been chosen so as to motivate a possible experiment in which a gate voltage sweep on the same device can span all characteristic regions of the steep, the intermediate and the smooth confinement.

In conclusion, we have demonstrated that a quantum wire in the IQHE regime exhibits a hysteretic behavior and spin phase transitions arising as an interplay between a confining potential, the Coulomb interactions and the exchange interaction.

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